

ملخص درس الحساب المثلثي

$$\cos^2 a = \frac{1 + \cos 2a}{2}$$

$$\sin^2 a = \frac{1 - \cos 2a}{2}$$

$$\cos^2 a + \sin^2 a = 1$$

$$1 + \tan^2 a = \frac{1}{\cos^2 a}$$

$$\cos 2a = \cos^2 a - \sin^2 a$$

$$\cos 2a = 2 \cos^2 a - 1$$

$$\cos 2a = 1 - 2 \sin^2 a$$

$$\sin 2a = 2 \sin a \cos a$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

$$\sin(a-b) = \sin a \cos b - \cos a \sin b$$

تحويل مجموع إلى جداء

$$\cos p + \cos q = 2 \cos\left(\frac{p+q}{2}\right) \cos\left(\frac{p-q}{2}\right)$$

$$\cos p - \cos q = -2 \sin\left(\frac{p+q}{2}\right) \sin\left(\frac{p-q}{2}\right)$$

$$\sin p + \sin q = 2 \sin\left(\frac{p+q}{2}\right) \cos\left(\frac{p-q}{2}\right)$$

$$\sin p - \sin q = 2 \cos\left(\frac{p+q}{2}\right) \sin\left(\frac{p-q}{2}\right)$$

تحويل جداء إلى مجموع

$$\cos a \cos b = \frac{1}{2} [\cos(a+b) + \cos(a-b)]$$

$$\sin a \sin b = -\frac{1}{2} [\cos(a+b) - \cos(a-b)]$$

$$\sin a \cos b = \frac{1}{2} [\sin(a+b) + \sin(a-b)]$$

$$\cos a \sin b = -\frac{1}{2} [\sin(a+b) - \sin(a-b)]$$

$$\tan(2a) = \frac{2 \tan a}{1 - \tan^2 a} \quad \text{و} \quad \tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b} \quad \text{و} \quad \tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

بوضع :  $t = \tan\left(\frac{x}{2}\right)$  لدينا :  $\tan x = \frac{2t}{1-t^2}$  و  $\cos x = \frac{1-t^2}{1+t^2}$  و  $\sin x = \frac{2t}{1+t^2}$

حيث :  $\sin \alpha = \frac{b}{\sqrt{a^2+b^2}}$  و  $\cos \alpha = \frac{a}{\sqrt{a^2+b^2}}$   $a \cos x + b \sin x = \sqrt{a^2+b^2} \cos(x-\alpha)$

|          |           |           |           |                    |                     |
|----------|-----------|-----------|-----------|--------------------|---------------------|
|          | $-x$      | $\pi-x$   | $\pi+x$   | $\frac{\pi}{2}-x$  | $\frac{\pi}{2}+x$   |
| $\cos x$ | $\cos x$  | $-\cos x$ | $-\cos x$ | $\sin x$           | $-\sin x$           |
| $\sin x$ | $-\sin x$ | $\sin x$  | $-\sin x$ | $\cos x$           | $\cos x$            |
| $\tan x$ | $-\tan x$ | $-\tan x$ | $\tan x$  | $\frac{1}{\tan x}$ | $\frac{-1}{\tan x}$ |

|          |     |                      |                      |                      |                 |
|----------|-----|----------------------|----------------------|----------------------|-----------------|
|          | $0$ | $\frac{\pi}{6}$      | $\frac{\pi}{4}$      | $\frac{\pi}{3}$      | $\frac{\pi}{2}$ |
| $\sin x$ | $0$ | $\frac{1}{2}$        | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | $1$             |
| $\cos x$ | $1$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$        | $0$             |

لكل  $x$  من  $\mathbb{R}$   $-1 \leq \sin x \leq 1$  ,  $-1 \leq \cos x \leq 1$   
 لكل  $x$  من  $\mathbb{R}$   $\cos^2 x + \sin^2 x = 1$   
 $1 + \tan^2 x = \frac{1}{\cos^2 x}$   $\mathbb{R} - \left\{ \frac{\pi}{2} + k\pi \right\}$

لكل  $x$  من  $\mathbb{R}$   $\cos(x + 2k\pi) = \cos x$   
 لكل  $k \in \mathbb{Z}$   $\sin(x + 2k\pi) = \sin x$

$\tan(x + k\pi) = \tan x$   $\tan x = \frac{\sin x}{\cos x}$

$k \in \mathbb{Z}$   $\cos x = \cos y$  تكافئ : أو  $\begin{cases} x = y + 2k\pi \\ x = -y + 2k\pi \end{cases}$

$k \in \mathbb{Z}$   $\sin x = \sin y$  تكافئ : أو  $\begin{cases} x = y + 2k\pi \\ x = (\pi - y) + 2k\pi \end{cases}$

$k \in \mathbb{Z}$   $\tan x = \tan y$  تكافئ :  $x = y + k\pi$